String Stability Preserving Adaptive Spacing Policy for Handling Saturation in Heterogeneous Vehicle Platoons ⋆

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Abstract: The saturation problem in heterogeneous vehicle platoons is considered. With leader and predecessor following control architecture, actuator saturation in a vehicle causes its followers to collide due to their link with the faster leader vehicle. A modification of the actual leader and predecessor following control method is proposed that automatically detects the saturation of other vehicles ahead, and reconfigures the tracking strategy to avoid collisions. The method is based on spacing policy estimation with the help of a virtual predecessor vehicle model. The proposed controller does not require additional measurements or extra communication, and is designed to satisfy string stability requirements. The results are verified by simulation examples.

Keywords: Platooning, String stability, Saturation, Collision, Leader and predecessor following, Adaptive spacing policy

1. INTRODUCTION

A feasible solution to the problems of overloaded road network is to exploit available capacities more efficiently. A vast part of developments in the automotive industry is directed toward an automated and cooperative transportation network where self-driving cars with communication- and sensor-based technologies can deliver improved safety, mobility, and reduced fuel consumption. One basic tool to achieve these goals is automated platooning. A platoon is a group of autonomous vehicles organized to cooperate with each other aiming to safely follow the platoon leader with smallest possible inter-vehicle gaps, see Alvarez and Horowitz (1997); Tsugawa et al. (1997); Bergenhem et al. (2010); Alfraheed et al. (2011); Kunze et al. (2009).

String stability (SS) of the platoon is the main issue that must be guaranteed when designing the platoon control system. Roughly speaking, SS means that tracking errors do not amplify backwards along the vehicle string. Several definitions of SS exist depending on the source of the trajectory perturbation like nonzero initial conditions, change in the reference input or disturbances, Sheikholeslam and Desoer (1990); Swaroop and Hedrick (1996); Ploeg et al. (2014). If only the spatial evolution of some performance outputs is to be examined in a homogeneous platoon, as effect of a single disturbance at the lead vehicle, it is common to prescribe that the sequence of worst case signals of interest be monotonously decreasing. For heterogeneous platoons with diverse mass/power ratio this monotony requirement is too stringent as it was demonstrated by Shaw and Hedrick (2007).

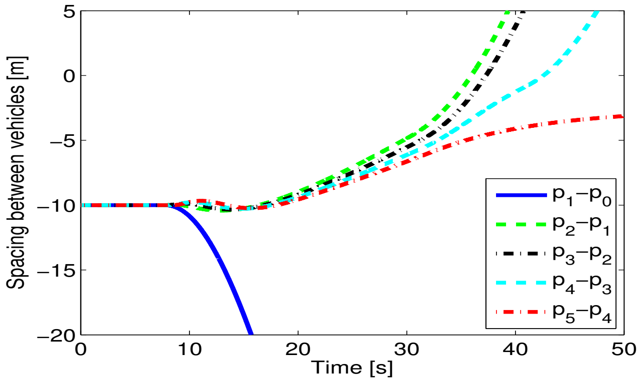

Fig. 1. Platoon simulation with a 5 degree uphill starting at 10s. Saturation of the first follower (vehicle 1) results in the collision of vehicles 1-4 ($p_i - p_{i-1} > 0$).

A common platoon control architecture that guarantees SS is leader and predecessor following (LPF) where the driving/braking force depends on the motion of both the leader and the predecessor vehicle. String stability and even safety are seriously compromised in LPF strategies if there are actuator saturations in the vehicles. If a vehicle between the ego and the leader vehicle cannot keep space due to its high mass/power ratio, the non-saturating ego vehicle may collide to its predecessor, see Mihály and Gáspár (2012) and Fig. 1.

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Some authors propose to limit the type of vehicles that can be organized in one group, but this approach would limit the applicability of platooning and would require a large burden of organization. Jovanovic et al. (2004) provided a method for computing the set of allowable initial conditions and corresponding feedback-gain constraints for avoiding saturation in each vehicles. Warnick and Rodriguez (1994, 2000) considered the performance degradation problem when controllers with integral action wind up in a transient period, and proposed anti-windup solutions for the same vehicle. They assumed that all vehicles can reach the same speed limit, so the saturation problem ceased after the windup transient. In contrast, Mihály and Gáspár (2012) considered a platoon of vehicles with diverse mass/power ratio and with extensive simulation studies illustrated the problem on a hilly terrain. They examined three methods to avoid collision: 1.) a priori organization of the vehicles according to decreasing mass/power ratio is not feasible due to dynamically changing conditions; 2.) informing the leader vehicle to slow down to fit the weakest vehicle in the group degrades performance of the whole group and avoiding collision in a longer platoon is not guaranteed; 3.) avoiding collision by breaking up the platoon to mini-platoons requires that the saturating vehicle informs all of its followers about saturation and about the taking over of leadership. This approach requires continuous organization of breaking up and then re-union of platoons.

In this paper we propose an automatic mechanism for the ego vehicle to deal with saturation of other vehicles ahead. Using this method by every member of the platoon, collisions are avoided while SS is preserved. In long uphill roads, the breaking up the platoon is unavoidable. Overtaking maneuvers and reorganization of the vehicle ordering is an issue that can be carried out independently of our proposed method.

Vehicle models with actuator saturation and the control structure are presented in Section 2. Detection of saturation in other vehicles and the spacing policy reconfiguration method are discussed in Section 3. The design method for achieving SS and simulation results are presented in Sections 4 and 5, respectively.

2. VEHICLE MODELING

In this section the models for longitudinal controller design as well as the platoon controllers are presented.

2.1 Vehicle models

The longitudinal dynamics of a vehicle is a highly nonlinear and hybrid dynamics due to the engine-speed dependent motor torque characteristics, gear dependent torque transmission and the gear change process. References Hedrick et al. (2001) and Gerdes and Hedrick (1997) present models for the longitudinal vehicle dynamics and nonlinear controllers that may serve as low level controllers tracking a speed or acceleration reference signal. The closed-loop system can be well approximated in certain limits by the following simple linear dynamics with two integrators and an actuator saturation model

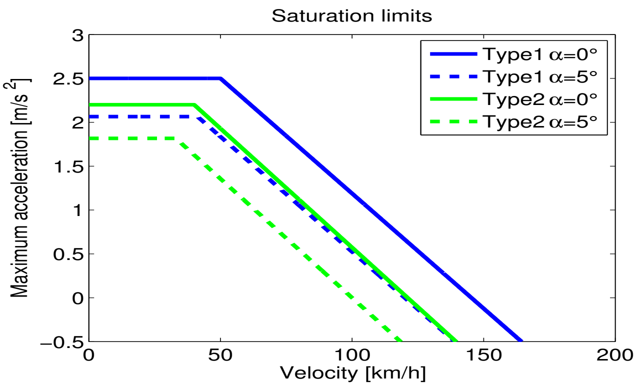

Fig. 2. Saturation limits for two types of vehicles: Type 1 vehicle has lower mass/power ratio than Type 2 vehicle. Vehicle acceleration is limited by speed and road slope dependent limits.

$$\dot{p}_i(t) = v_i(t) \tag{1}$$

$$\dot{v}_i(t) = a_i(t) \tag{2}$$

$$\dot{a}_i(t) = -\frac{1}{\tau_i}a_i(t) + \frac{1}{\tau_i}u_i^{sat}(t) \tag{3}$$

where u_i denotes the acceleration reference provided by the platoon controller, and τ_i is a constant parameter of the closed-loop dynamics. Let $H_i(s) = \frac{1}{\tau_i s+1}$ denote the transfer function of the linear actuator dynamics. The finite power of the vehicle is modeled by a speed and road-slope dependent nonlinear function $u_i^{sat}(t) = u_i^{sat}(u_i(t); v_i(t), \alpha(t))$ defined by

$$u_i^{sat}(u; v, \alpha) = \begin{cases} u & \text{if } v < v_z \text{ and } u \le a_{max} \\ a_{max} & \text{if } u > a_{max} \text{ and } v < v_z \\ \dfrac{v - v_{max}}{v_z - v_{max}}a_{max} & \text{if } v \ge v_z \end{cases} \tag{4}$$

$$a_{max}(\alpha) = a_{max,0}(1 - 2\sin(\alpha)) \tag{5}$$

$$v_{max}(\alpha) = v_{max,0}(1 - 2\sin(\alpha)) \tag{6}$$

$$v_z(\alpha) = v_{z,0}(1 - 2\sin(\alpha)) \tag{7}$$

The saturation limit function is plotted in Fig. 2.

2.2 Controller structure

It is assumed that the spacing errors, $e_{i,0}$, $e_{i,i-1}$, their derivatives and accelerations a_0, a_{i-1}, are available at vehicle i. The goal of each distributed spacing controllers is to follow a linear combination of the motion of reference vehicles, 0 and $i-1$. The controller is defined in the Laplace domain as follows

$$u_i(s) = K_{0,i}(s)\begin{bmatrix} a_0(s) - a_i(s) \\ R_{i,0}(s) \end{bmatrix} + K_{1,i}(s)\begin{bmatrix} a_{i-1}(s) - a_i(s) \\ R_{i,i-1}(s) \end{bmatrix} \tag{8}$$

where

$$K_{j,i}(s) = \left[\frac{K_{a,i}^j s^2 + K_{v,i}^j s + K_{p,i}^j}{s^2},\ K_{p,i}^j \right],\ j = 0, 1 \tag{9}$$

with appropriate constant parameters, $K_{a,i}^j, K_{v,i}^j, K_{p,i}^j \ge 0$. This control structure is common in LPF architectures, see Swaroop (1994), except that the desired spacing functions, $R_{i,j}$ are expressed as inputs. For an LPF controller with constant spacing policy, $R_{i,j}(t) = L_{i,j}$ are constants.

3. ADAPTIVE SPACING POLICY

An adaptive spacing policy has been recently developed by Rödönyi and Szabó (2016) for vehicles in ad hoc platoons where mixed human driven and autonomous vehicles are allowed to follow each other. In those heterogeneous platoons an autonomous vehicle that applies an LPF control strategy faces similar problems to the problem with saturating predecessors: preceding vehicles do not keep the presumed spacing policy and due to the link with the leader, the ego vehicle runs down its predecessor. The solution proposed by Rödönyi and Szabó (2016) was to estimate a virtual spacing policy of a virtual predecessor vehicle that can be computed online based on already available measurements. In that paper SS was not considered, but there remained design freedom in the parameters to cope with the SS issue.

In this paper the adaptive spacing policy by Rödönyi and Szabó (2016) is adopted for the platooning problem with saturating actuators. The unknown input observer for the virtual predecessor vehicle is modified here, so that a relatively simple design procedure leads to SS.

3.1 Consistency of spacing policies

From the point of view of the ego vehicle (vehicle i) it is irrelevant that a predecessor (any follower vehicle ahead, $i = 1, 2, ..., i-1$) deliberately follows the leader vehicle according to a different policy (mixed traffic) or due to some fault, or due to actuator saturation. The result is that the spacing policies are inconsistent.

Let $R_{i,j}$ denote the desired spacing of vehicle i with respect to vehicle j, for all j which is defined by the control strategy. Let vehicle i be controlled by a LPF strategy, so $j = 0$ or $j = i-1$. The control goal of vehicle i is to drive its spacing errors

$$e_{i,j}(t) = p_j(t) - p_i(t) - R_{i,j}(t), \quad \forall j \in \{0, i-1\} \tag{10}$$

to zero. This is only possible when

$$R_{i-1,0}(t) + R_{i,i-1}(t) = R_{i,0}(t), \tag{11}$$

is satisfied, which is the definition of consistency of spacing policies between vehicles 0, $i-1$, and i, see Rödönyi and Szabó (2016).

For vehicle i the spacing policy of the predecessor $R_{i-1,0}(t)$ is not known in an ad hoc (mixed) platoon. Similarly, the actual "spacing policy" $R_{i-1,0}(t)$, which is now determined by the actuator saturation, is not known for the control of vehicle i. The goal is to estimate the actual spacing policy of the predecessor and modify the local policy $R_{i,0}(t)$ to satisfy (11). If we chose a constant spacing with respect to the predecessor, say $R_{i,i-1}(t) = L_{i,i-1}$, then the adaptive spacing policy with respect to the leader vehicle is given by

$$R_{i,0}(t) = L_{i,i-1} + \hat{R}_{i-1,0}(t) \tag{12}$$

where $\hat{R}_{i-1,0}(t)$ is the virtual spacing policy of the virtual predecessor vehicle with respect to the leader.

3.2 Virtual Predecessor

A virtual predecessor vehicle is a virtual vehicle model whose acceleration, speed and position coincides with those of the true predecessor vehicle. It has two type of inputs. 1.) acceleration of the leader, a_0, relative speed, $v_0 - v_{i-1}$, and relative position, $p_0 - p_{i-1}$ - all measurements are available for vehicle i; 2.) spacing demand $\hat{R}_{i-1,0}$ which is an unknown input, but it drives the virtual vehicle to move as if glued together with the true predecessor vehicle.

The choice of the virtual predecessor model influences the distribution of the distance $p_0 - p_{i-1}$ between a "deliberate" part, $\hat{R}_{i-1,0}$, and a spacing error part, $\hat{e}_{i-1,0} = p_0 - p_{i-1} - \hat{R}_{i-1,0}$. The choice influences the spacing error of the ego vehicle $e_{i,i-1}$ and also the freedom left for designing a controller for vehicle i to achieve SS.

The spacing policy of the predecessor vehicle can be estimated based on the inversion of a virtual predecessor vehicle model. Let the virtual predecessor vehicle be a system with predecessor following control architecture defined by the equations

$$a_v(s) = H_v(s)u_v(s) \tag{13}$$

$$u_v(s) = K_v(s)(a_0(s) - a_v(s)) + K_p^v \hat{R}_{i-1,0}(s) \tag{14}$$

$$K_v(s) = \frac{K_a^v s^2 + K_v^v s + K_p^v}{s^2} \tag{15}$$

$$H_v(s) = \frac{1}{\tau_v s + 1} \tag{16}$$

where subscript v refers to "virtual", and vehicle index $i-1$ is omitted for brevity of notation. The closed-loop virtual model reveals

$$a_v(s) = [\, V_1(s),\ V_2(s) \,] \begin{bmatrix} a_0(s) \\ \hat{R}_{i-1,0}(s) \end{bmatrix} \tag{17}$$

$$V_1(s) = \frac{H_v(s)K_v(s)}{1 + H_v(s)K_v(s)} \tag{18}$$

$$V_2(s) = \frac{H_v(s)K_p^v}{1 + H_v(s)K_v(s)}. \tag{19}$$

Since the system is controllable from input $\hat{R}_{i-1,0}$, there exists a function $\hat{R}_{i-1,0}(t)$ such that the requirements $a_v \equiv a_{i-1}$, $v_v \equiv v_{i-1}$, $p_v \equiv p_{i-1}$ can be achieved, i.e., we have from (17)

$$a_{i-1}(s) = V_1(s)a_0(s) + V_2(s)\hat{R}_{i-1,0}(s) \tag{20}$$

for an appropriate virtual spacing policy $\hat{R}_{i-1,0}(.)$.

3.3 Approximation of Virtual Spacing Policy

Using dynamic inversion for system V_2 would require the use of acceleration derivatives, which we would like to avoid, therefore an approximate inverse is applied instead. To derive an approximate inverse, multiply (20) by $Q(s) = \tau_q s + 1$, with design parameter $\tau_q > 0$,

$$Q(s)a_{i-1}(s) = Q(s)V_1(s)a_0(s) + Q(s)V_2(s)\hat{R}_{i-1,0}(s) \tag{21}$$

and define $S(s) = (Q(s)V_2(s))^{-1}$ to obtain

$$\hat{R}_{i-1,0}(s) = S(s)Q(s)(a_{i-1}(s) - V_1(s)a_0(s)). \tag{22}$$

Since $S(s)Q(s)$ is not a proper system and the computation of $\hat{R}_{i-1,0}$ would require the derivation of acceleration measurements, we define a filtered spacing demand,

$$R^Q_{i-1,0}(s) = S(s)(a_{i-1}(s) - V_1(s)a_0(s)). \tag{23}$$

Using $R^Q_{i-1,0}$ instead of $\hat{R}_{i-1,0}$ have some consequences on the dynamics of vehicle i in exchange for giving up the exact much of $a_v \equiv a_{i-1}$. To see this, combine (17) with (23),

$$\begin{aligned} Q(s)a_v(s) &= Q(s)V_1(s)a_0(s) \\ &+ Q(s)V_2(s)S(s)(a_{i-1}(s) - V_1(s)a_0(s)), \end{aligned} \tag{24}$$

which implies

$$a_v(s) = \frac{1}{\tau_q s + 1} a_{i-1}(s) + \frac{\tau_q s}{\tau_q s + 1} V_1(s)a_0(s). \tag{25}$$

In steady-state, a_v equals to a_{i-1} and at small frequencies it approximates a_{i-1} well. The high frequency component of measurements a_{i-1}, a_0, typically the noise content, is filtered out. Depending on the choice of τ_q the leader motion has a disturbing effect through band-pass filter $\frac{\tau_q s}{\tau_q s+1} V_1(s)$. A unit step change in the acceleration of the leader while the predecessor is saturating with $v_{i-1}(t) = const$ results in a steady state speed difference of τ_q between the predecessor and the virtual vehicle, and this will increase the spacing error, $e_{i,i-1}$ of vehicle i.

The control scheme with the virtual spacing policy estimation

$$R^Q_{i-1,0}(s) = E(s) \begin{bmatrix} a_0(s) \\ a_{i-1}(s) \end{bmatrix}, \; E(s) = [\, -S(s)V_1(s), \; S(s)\,] \tag{26}$$

is illustrated in Fig. 4.

4. DESIGN FOR STRING STABILITY

The design for SS of the adaptive spacing controller can be started from the nominal case without spacing adaptation. Then the controller is extended with the unknown virtual spacing policy estimator and a new feedback-loop.

4.1 Nominal Controller Design

One goal of the control design is to drive the spacing errors to zero in steady state and the second is to guarantee SS for attenuating the transient processes. The first goal is achieved by the control structure shown in Fig. 3 provided that the spacing policies are consistent in the platoon, see Rödönyi and Szabó (2016). The design procedure for SS is detailed in Rödönyi (2015). The basic steps are the following.

First design a good reference following controller, $K_i(s)$,

$$u_i(s) = K_i(s)(a_{\rho,i}(s) - a_i(s)) \tag{27}$$

$$a_{\rho,i}(s) = \rho_i a_0(s) + (1-\rho_i)a_{i-1}(s) \tag{28}$$

$$K_i(s) = \frac{K_{a,i}s^2 + K_{v,i}s + K_{p,i}}{s^2}, \tag{29}$$

to track reference $a_{\rho,i}(s)$. The closed-loop system $T_n(s) = \frac{K_i(s)H_i(s)}{1+K_i(s)H_i(s)}$ has the following properties, $T_n(0) = 1$ and $\|T_n\|_\infty > 1$ by Bode complementary sensitivity integral theorem, Seiler et al. (2004).

The condition for SS is that the closed-loop gain from a_{i-1} to a_i is less than one for all $\omega \geq 0$, see Rödönyi (2015), i.e., $\|(1-\rho_i)T_n\|_\infty < 1$, which can be satisfied by an appropriate choice for ρ_i.

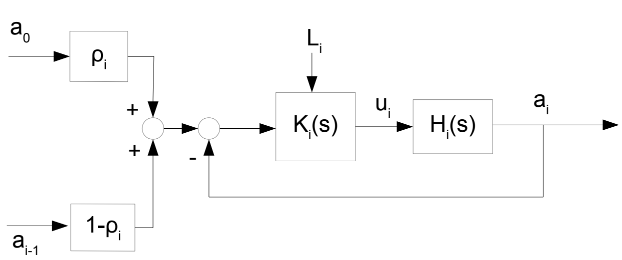

Fig. 3. Leader and predecessor following control architecture at vehicle i *without* spacing policy adaptation

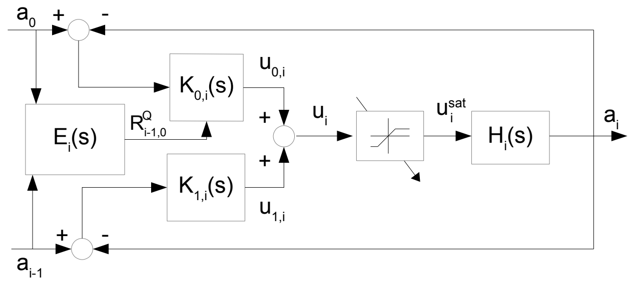

Fig. 4. Leader and predecessor following control architecture at vehicle i *with* spacing policy adaptation

4.2 Adaptive Controller Design

The goal of the spacing-adaptive control design is to recover the normal operation when no saturation occurs, and in case of saturation, the collisions are avoided, and the spacing errors remain small and bounded.

The design parameters are the following, $K_{1,i}(s)$, $K_{0,i}(s)$, $H_v(s), K_v(s), \tau_q, \tau_v$. It will be shown by simulation in the next section that by using the following simple design procedure the desired goals can be achieved.

1.) Let us fix $\tau_q > 0$. Let $K_{1,i}(s) = (1-\rho_i)K_i(s)$, where $K_i(s)$ is designed in the previous subsection.

2.) The choice $K_{0,i}(s) = \rho_i K_i(s)$ would recover the nominal case. Instead, let $K_{0,i}(s) = \rho_i K_v(s)$, where $K_v(s)$ is still unknown, and let $\tau_v = \tau$.

3.) If the set of closed-loop poles of the nominal system are denoted by $\lambda(T_n)$, then let the virtual predecessor model V_1 is designed to have $\lambda(V_1) = c^v_{bw}\lambda(T_n)$, i.e., the bandwidth and the poles of the nominal transfer function is multiplied by a factor c^v_{bw}. Thus $K_v(s)$ is given by the solution of a pole-placement problem.

It is interesting to note that the SS condition changed for the adaptive architecture. From Fig. 5 it can be seen that the transfer function mapping a_0 to a_i has a zero in 0. Therefore, it is no longer required for ensuring SS that $|T_{a_i,a_{i-1}}(0)| < 1$, where

$$T_{a_i,a_{i-1}}(s) = \frac{H_i(s)(K_{1,i}(s) + S(s)K^0_{p,i})}{1 + H_i(s)(K_{1,i}(s) + K_{0,i}(s))}. \tag{30}$$

String stability is guaranteed if $|T_{a_i,a_{i-1}}(j\omega)| < 1$, $\forall \omega > 0$, and $|T_{a_i,a_{i-1}}(0)| \leq 1$, see Rödönyi (2015).

String stability condition $\|T_{a_i,a_{i-1}}\|_\infty \leq 1$ can be achieved by increasing the bandwidth factor c^v_{bw}. With increasing

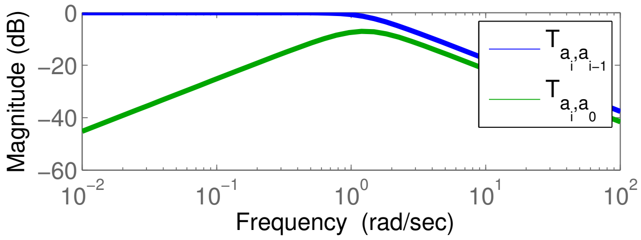

Fig. 5. Magnitude functions of the closed-loop system in Fig. 4

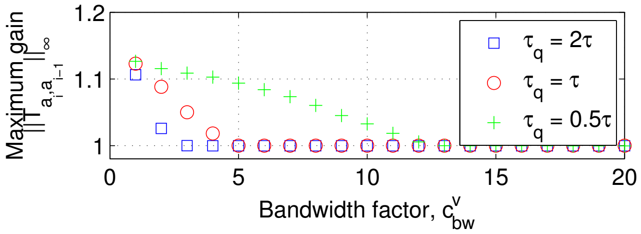

Fig. 6. Trade-off between fast spacing adaptation (small τ_q) and required control bandwidth for achieving SS ($\|T_{a_i,a_{i-1}}\|_\infty \leq 1$)

c^v_{bw}, however, the control action $u_{0,i}$ is also increasing, therefore it is worth keeping c^v_{bw} as small as possible.

Unfortunately, there is a trade-off also between SS and the convergence of the spacing policy estimation. The worst-case gain $\|T_{a_i,a_{i-1}}\|_\infty$ is plotted for several values of c^v_{bw} and τ_q in Fig. 6. Smaller τ_q results in faster convergence of $R^Q_{i-1,0}$ to $\hat{R}_{i-1,0}$, but also higher c^v_{bw}, and so higher control effort is necessary to satisfy the SS condition $\|T_{a_i,a_{i-1}}\|_\infty \leq 1$.

5. SIMULATION RESULTS

In this section the proposed adaptive spacing platoon control algorithm is verified by simulation examples. The vehicle and controller parameters are the same for all vehicles, $\tau_i = 0.7$, $K^1_{a,i} = 0.91$, $K^1_{v,i} = 0.4$, $K^1_{p,i} = 1.58$, $\rho_i = 0.1281$, $\tau_q = \tau_i$, $c^v_{bw} = 5$, $K^0_{a,i} = 4.55$, $K^0_{v,i} = 13.5$, $K^0_{p,i} = 10$. The vehicles differ in the saturation limits given by $a_{max,0} = 2.5m/s^2$, $v_{max,0} = 145.33km/h$, $v_{z,0} = 50km/h$ for Type 1 vehicle, and $a_{max,0} = 2.2m/s^2$, $v_{max,0} = 122.11km/h$, $v_{z,0} = 40km/h$ for the vehicle with higher mass/power ratio.

The simulation scene is the following. The platoon is traveling on level road at speed 115km/h. Then at $t = 10s$ the leader reaches an uphill of $\alpha = 5$ degrees. The separation distance is 10m. All vehicles but the first follower are of Type 1. So the first follower saturates on the uphill, its speed reduces to 100km/h. Fig. 1 shows the case with standard leader and predecessor following platoon controllers on each vehicles. Vehicle 1 drops behind while the other followers are pulled by the leader resulting in collisions ($p_i(t) - p_{i-1}(t) > 0$).

The effect of spacing adaptation is shown in Fig. 7. The leader keeps its speed of 115km/h until $t = 30s$, while followers $i > 1$ keep space with vehicle 1. The spacing error increased to about 2m. Then the leader accelerates with $a_0 = 1m/s^2$ between $t \in [35s, 45s]$, which increases

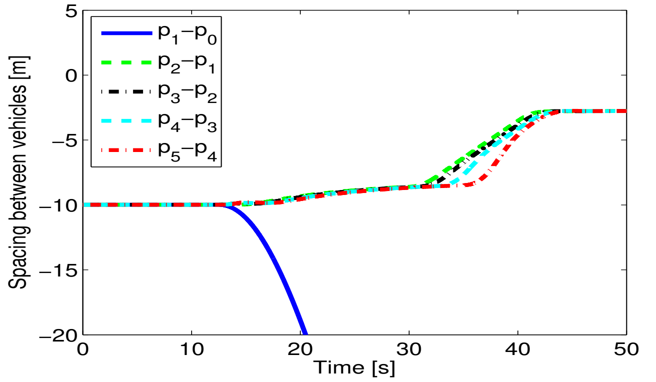

Fig. 7. Spacing in case of the proposed adaptive spacing policy controllers

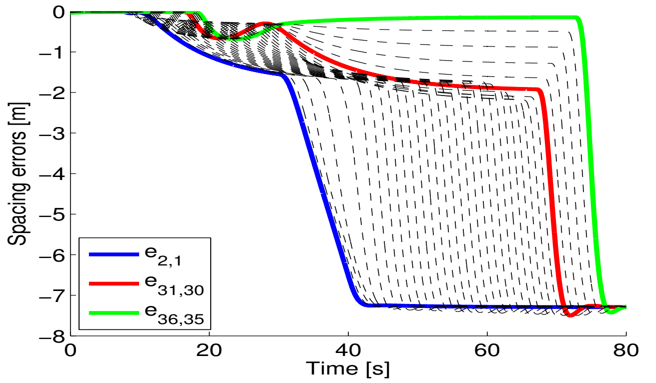

Fig. 8. String stability during saturation on uphill and leader acceleration

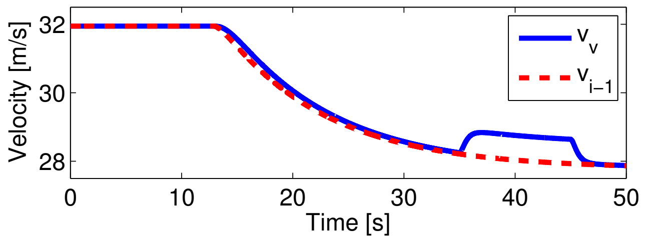

Fig. 9. Speed difference of virtual and true predecessor vehicles

further the spacing errors of the followers according to (25), see Fig. 8. This performance degradation is due to the approximation error $R^Q_{i-1,0}(t) - \hat{R}_{i-1,0}(t)$. Fig. 9 shows the speed difference between the predecessor and the virtual vehicle. Nevertheless, the collisions are still avoided. The spacing errors in Fig. 8 are shown in a longer time period to see the SS of the formation.

To see how the nominal performance is recovered the end of the scene is modified as follows, see Fig. 10. The leader accelerates between $t = 30s$ and $t = 35s$ with $a_0(t) = 1m/s^2$ then decelerates between $t = 45s$ and $t = 50s$ with $a_0(t) = -1m/s^2$. The road is again horizontal from $t = 38s$. The followers reach the leader at about $t = 183s$. After that time the spacing errors reduce to zero, so the nominal operation is recovered.

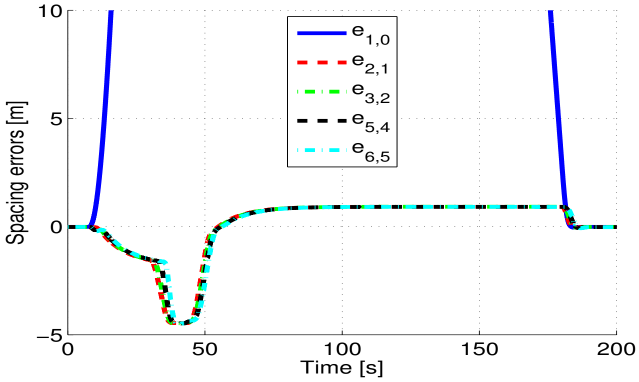

Fig. 10. Recovering nominal performance

6. CONCLUSION

An adaptive spacing policy based solution is proposed to handle the problem caused by actuator saturation of vehicles in a platoon of diverse mass/power ratio. In this approach the virtual spacing policy of the predecessor vehicle is only approximated, and the gluing together of the virtual and the predecessor vehicle is not ensured. This has a negative consequence on the tracking performance during saturation of vehicles, nevertheless collisions are avoided, string stability is guaranteed, and nominal performance is recovered after saturation effects ceased.

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